

THERMCAT – On-Line Tool for Simulation of Current Transfer to Thermionic Cathodes of High-Pressure Arc Discharges

http://www.arc_cathode.uma.pt/tool/index.htm

Tutorial

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The 1st version of the code was released on June 11, 2005.

The 2nd version of the code was released on May 15, 2007.

The 3rd version of the code was released on April 30, 2009.

This tutorial describes the 3rd version of the code with the changes made up to the above date.

What is new in the 3rd version:

- The possibility has been introduced of calculation, in addition to the low-current section of the diffuse mode, also of other axially symmetric modes of current transfer (the high-current section of the diffuse mode and axially symmetric spot modes);
- A module has been added for calculation of bifurcation points positioned on axially symmetric modes of current transfer, in particular, of the limit of stability of the diffuse mode;
- The concept of the code has changed: now it can be used not only for simulation of axially symmetric spot modes of current transfer from high-pressure plasmas to cylindrical thermionic cathodes, but also as a tutorial on finding multiple solutions describing different modes of current transfer.

The objective of this code is two-fold. The first one is to provide a fast and robust tool for simulation of diffuse and axially symmetric spot modes of current transfer from high-pressure plasmas to cylindrical thermionic cathodes in a wide range of arc currents, cathode materials and dimensions, and plasma compositions. The second objective is methodological. Nowadays, it is universally understood that different modes of current transfer to thermionic cathodes are described by multiple solutions that must exist under certain discharge conditions in an adequate theoretical model. Unfortunately, the concept of multiple solutions, while being quite common in many areas of theoretical physics, is not equally widespread in applied physics. Therefore, we tried to offer to applied physicists and engineers working in the field a tutorial that would help them to make themselves comfortable with multiple solutions describing different modes of current transfer to thermionic cathodes. It should be stressed that the experience gained with this tutorial will facilitate work with other solvers, including commercial software like COMSOL Multiphysics or ANSYS: difficulties encountered in finding multiple solutions by means of any efficient solver are the same.

The code employs the model of nonlinear surface heating (e.g., [1]), is based on the equations summarized in Appendix to the paper [2], and consists of three modules. The first module provides a solution describing the near-cathode plasma layer. The second module provides a 2D solution of the heat conduction equation in the cathode body under the assumption of axial symmetry. The third module is dedicated to calculation of bifurcation points positioned on the axially symmetric mode.

We have worked a lot with this code under very different conditions. However, we have not attempted to fine-tune it for all conditions imaginable. Hence, **we cannot guarantee its applicability to all imaginable conditions.**

You will need to perform **three steps**. The first one is definition of control parameters for simulations. The second step is generation of a starting-point solution. The third and final step is execution of simulations in a wide range of arc currents.

The outline of this tutorial is as follows. Sections 1 to 3 are dedicated to, respectively, above-described steps 1 to 3 and provide a complete description of **basic features** of the code. If you are interested in simulation of only diffuse mode of current transfer, you do not have to read further and may stop here. Sections 4 and 5 are dedicated to **advanced features**. The calculation of multiple solutions describing both diffuse and axially symmetric spot modes is explained in Section 4. Section 5 is concerned with finding the limit of stability of the diffuse mode and bifurcation points at which other modes, in particular, 3D spot modes, branch off from the axially symmetric modes. Again, each of these sections provides a complete description of the corresponding feature and these sections may be read independently one from the other. The last part of the tutorial describes a few additional details needed in the case where you run on your computer a **Windows version of the code** (instead of running the code via Internet on a server of Universidade da Madeira, which is the case described in Sections 1 to 5).

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Part A. Basic features

1. Step 1: Specifying input parameters

Plasma-producing gas

If you wish to work with a pure monoatomic plasma-producing gas, just enter in this field the chemical symbol, e.g., He. The first character must be non-blanc. Please observe the difference between upper and lower-case letters. The code will use atomic parameters and other data needed for calculation of the near-cathode plasma layer [2] (the rate constants of direct and stepwise ionization of atoms by electron impact and cross-section for momentum transfer in elastic collisions ion-atom) for the gas being considered, taken from the internal database. This version of the database includes, but is not limited to, He, Ne, Na, Ar, Cu, Kr, Xe, Cs, and Hg; if the gas you wish to work with is not in the database, the code will issue an error message and terminate. This version of the database allows also working with the following mixtures: air (entry: ai), mixture Na-Hg (entry: NH), mixture Cs-Hg (entry: CH), plasmas of mercury with addition of metal halides (entry: MH; the percentage of additives is assumed to be small), plasmas of xenon with addition of metal halides (entry: XH; the percentage of additives is assumed to be small).

Plasma pressure

Enter pressure of the plasma (in bars).

Cathode material

Enter in this field the chemical symbol of metal which the cathode is made of, e.g., W. The first character must be non-blanc; for example, 'W' is permitted while ' w' will cause an error message and the code will terminate. Observe the difference between upper and lower-case letters. The code will use the temperature-dependent thermal conductivity, the temperature-dependent hemispherical total emissivity, and thermionic emission properties for the specified cathode material, taken from the internal database. This version of the database includes, but is not limited to, W, Mo, Hf, Fe, Nb, and Zr; if you specified a metal which is not in the database, the code will issue an error message and terminate.

Cathode radius

Enter radius of the cathode (in m).

Cathode height

Enter height of the cathode (in m).

Temperature of the base

Enter temperature of the base of the cathode (in K).

Radiation

Enter .t. (true) if you wish to take into account the cooling of the surface of the cathode by radiation. In this case, the (net) energy flux from the plasma to the cathode surface will be evaluated as the difference between the energy flux transported to the surface by the ions and the electrons and the radiative cooling of the cathode surface, evaluated in terms of T_w and the temperature-dependent hemispherical total emissivity of the cathode material taken from the internal database. Enter .f. (false) if you wish to neglect the radiative cooling. In this case, only the energy flux transported by the ions and the electrons will be taken into account.

Insulated lateral surface

Normally, this parameter is set equal to .f.. If you set this parameter equal to .t., then densities of the electric current and the energy flux from the plasma to the lateral surface of the cathode are set equal to zero, as well as cooling of the lateral surface by radiation. In other words, .t. corresponds to the case where the lateral surface of the cathode is electrically and thermally insulated. (This case is of theoretical interest because the diffuse-mode solution in this case is one-dimensional.)

Thermionic emission parameters will be taken from the internal database

The code evaluates thermionic emission of electrons from the cathode by means of the Richardson-Dushman formula. The formula involves two parameters: the work function and the pre-exponential factor. Leave the default value Yes in this field if you want the code to employ (constant) values of these parameters for the specified cathode material, taken from the internal database. Choose the value No if you want to specify these values by yourself. (This option is particularly useful while working with doped materials, e.g., thoriated tungsten.) A further option, which is available in the case where the cathode material is tungsten and the plasma-producing gas is one of the mixtures Na-Hg or Cs-Hg, is to make the code evaluate the work function taking into account its variation owing to the formation of a monolayer of alkali metal atoms on the surface [3].

Work function

(This field is present on the screen if you entered No in the field Thermionic emission parameters will be taken from the internal database.) Enter here the work function you are going to use (in eV).

Pre-exponential factor in the Richardson-Dushman formula

(This field is present on the screen if you entered No in the field Thermionic emission parameters will be taken from the internal database.) Enter here the pre-exponential factor in the Richardson-Dushman formula you are going to use (in $A / m^2 K^2$).

Variability of the work function

(This field is present on the screen if you entered No in the field Thermionic emission parameters will be taken from the internal database.) Leave the default value .f. in this field if you want the code to employ the (constant) values of the emission parameters specified in the fields Work function and Pre-exponential factor in the Richardson-Dushman formula. Choose the value .t. if you want the code to evaluate the (variable) work function of tungsten covered with sodium (plasma-producing gas = NH) or cesium (plasma-producing gas = CH) atoms [3]. (Values that appear in the fields Work function and Pre-exponential factor in the Richardson-Dushman formula have no effect in this case.) Attention: if this field reads .t., then cathode should be W and gas either NH or CH, otherwise the code will issue an error message and terminate. Note that the modelling with account of variations of the work function requires special care, especially in the case of CH plasma.

Content of sodium

If your plasma-producing gas is NH, or MH, or XH, enter in this field the mole fraction of the element of sodium in the mixture. This field has no effect if your plasma-producing gas is neither NH nor MH nor XH.

Content of thallium

If your plasma-producing gas is MH or XH, enter in this field the mole fraction of the element of thallium in the mixture. This field has no effect if your plasma-producing gas is neither MH nor XH.

Content of dysprosium

If your plasma-producing gas is MH or XH, enter in this field the mole fraction of the element of dysprosium in the mixture. This field has no effect if your plasma-producing gas is neither MH nor XH.

Content of scandium

If your plasma-producing gas is MH or XH, enter in this field the mole fraction of the element of scandium in the mixture. This field has no effect if your plasma-producing gas is neither MH nor XH.

Content of cesium

If your plasma-producing gas is CH, or MH, or XH, enter in this field the mole fraction of the element of cesium in the mixture. This field has no effect if your plasma-producing gas is neither CH nor MH nor XH.

Content of zinc

If your plasma-producing gas is XH, enter in this field the mole fraction of the element of zinc in the mixture. This field has no effect if your plasma-producing gas is not XH.

Content of indium

If your plasma-producing gas is XH, enter in this field the mole fraction of the element of indium in the mixture. This field has no effect if your plasma-producing gas is not XH.

Content of thorium

If your plasma-producing gas is XH, enter in this field the mole fraction of the element of thorium in the mixture. This field has no effect if your plasma-producing gas is not XH.

Content of iodine

If your plasma-producing gas is XH, enter in this field the mole fraction of the element of iodine in the mixture. This field has no effect if your plasma-producing gas is not XH.

In the case of metal halide plasmas (MH) the code evaluates mole fraction of the element of iodine assuming that the metal halides are added to mercury in the form of molecules NaI, TlI, DyI₃, ScI₃, CsI. In other words, mole fraction of the element iodine is evaluated in terms of mole fractions of the elements Na, Tl, Dy, Sc, and Cs as $ZI = ZNa + ZTl + 3(ZDy + ZSc) + ZCs$. It is assumed that the only element of metal halide plasmas besides Na, Tl, Dy, Sc, Cs, and I is Hg, i.e., the code assumes $ZHg = 1 - (ZNa + ZTl + ZDy + ZSc + ZCs + ZI)$.

In the case of plasmas of xenon with addition of metal halides (XH), it is assumed that the only element of the plasmas besides Na, Tl, Dy, Sc, Cs, Zn, In, ZTh, and I is Xe, i.e., the code assumes $ZXe = 1 - (ZNa + ZTl + ZDy + ZSc + ZCs + ZZn + ZIn + ZTh + ZI)$.

Example

This tutorial refers to the following example, unless other conditions are specified. Suppose that you wish to simulate interaction of an atmospheric-pressure argon plasma with a tungsten cathode of the

radius of 1 mm and the height of 10 mm, with the temperature of 20⁰C at the externally cooled base of the cathode, without insulation on the lateral surface of the cathode, taking into account cooling of the cathode by radiation, and using values of the work function of tungsten and of the pre-exponential factor in the Richardson-Dushman formula from the internal database. In this case, you should specify:

```
Ar      ! plasma-producing gas
1       ! plasma pressure in bars
W       ! cathode material
0.001  ! cathode radius in m
0.010  ! cathode height in m
293    ! temperature of the cathode base in K
.t.     ! radiation
.f.     ! insulated lateral surface
Yes     ! Thermionic emission parameters will be taken from the
        ! internal database
```

Contents of the fields Content of sodium, Content of thallium, Content of dysprosium, Content of scandium, Content of cesium, Content of zinc, Content of indium, Content of thorium, Content of iodine are irrelevant.

When all relevant input parameters have been specified, press the `Submit` button to transfer the information to the server.

2. Step 2: Generating a starting-point solution

Normally, you will want to perform simulations for several values of the near-cathode voltage drop U . A solution for the first value should be generated separately. Go to the tab entitled `Step 2`. The first field on the left-hand side of the screen, `The default numerical grid will be used (recommended)`, serves to customize the numerical grid; ignore it for now (i.e., leave in this field the default value `Yes`).

You need to specify the near-cathode voltage drop U (in volts) which corresponds to the starting point desired; enter 15 into the corresponding field. Choose `Yes` in the field `Generating starting-point solution automatically`. You need to specify the number N_s of steps in which the starting-point solution will be generated; enter 2 into the corresponding field. Press the button `Start`. The following information appears in the window on the right-hand side of the screen:

```
s = 0.0000 (initial approximation)
           Tcen=3477           Tedge=3477
s = 0.5000
  1           3335           3475
  2           3358           3453
  3           3358           3453
  4           3358           3453
s = 1.0000
  1           3279           3428
  2           3282           3419
  3           3281           3419
  4           3281           3419
```

The starting-point solution is ready.

Here s is a transition parameter, $s = 0$ corresponding to the built-in initial approximation and $s = 1$ to a solution desired. You can see that the solution was successfully generated in 2 steps: there were 4

iterations with $s = 0.5$ and 4 iterations with $s = 1$. The printout includes the temperatures at the center and at the edge of the front surface of the cathode, T_{cen} and, respectively, T_{edge} . Both temperatures are equal to 3477K in the initial approximation. In the converged starting-point solution, T_{cen} equals 3281K and is somewhat lower than $T_{edge} = 3419K$; a feature which is typical for the diffuse mode at low currents and originates in the conditions for heat conduction cooling at the center being more favorable than those at the edge. Note that if T_{cen} exceeds T_{edge} in a converged starting-point solution, this is an indication that this solution describes not the diffuse mode of current transfer but rather a spot mode; see Section 4 of this tutorial.

Change N_s to 1 and press the button `Start` once again. You can see that you have obtained the same solution. Typically, lower values of N_s are suitable for thin cathodes, high ionization potential of plasma species and high work function. For example, if you try to perform the same procedure with $N_s = 2$ for a 1.6-mm radius cathode, the following information will appear on the screen:

```
s = 0.0000 (initial approximation)
      Tcen=3477      Tedge=3477
s = 0.5000
  1          632          5074
  2
```

The current iteration has been completed. The calculated temperature of some points of the front surface is below T_{col} . The code terminated.

Here T_{col} is the temperature of the base of the cathode, which is specified in the corresponding field in the tab `Step 1`. You can see that there were two iterations with $s = 0.5$. The temperature at the center of the front surface of the cathode after the first iteration was unusually low (632K). After the second iteration, the distortion of the surface temperature distribution became still stronger: there was at least one point on the front surface with a temperature below T_{col} , so the code was interrupted. Setting $N_s = 3$ will fix the problem. Typically, suitable values of N_s are between 1 and 10. Values of N_s as high as 1000 were required in some simulations of mercury-containing plasmas; however, the code has been improved since then and we are not sure whether values that high could be useful now.

There are physical limitations on the near-cathode voltage drop U . The current-voltage characteristic of the diffuse mode has a minimum, which for our basic example (the one specified at the end of Section 1) is 10.00V. Therefore, if you try to generate a starting-point solution for U below the above-mentioned minimum, you will not succeed. In fact, you should choose a starting point somewhat higher than the minimum. For example, if you try to perform the above-described procedure for $U = 10V$, the following information will appear on the screen:

```
Unable to generate an initial approximation for the
specified value of U. Try a higher U. The code terminated.
```

You can see that the iterations have not even started, i.e., the code is unable to provide an initial approximation for $U = 10V$. The same happens if you set $U = 11V$, however $U = 12V$ is fine.

The physically reasonable range of U is limited also from above: there is not much sense in running the code with U exceeding, say, a few hundred volts, and even these values may look too high to most people (in fact, such values are sometimes needed for analysis of transient effects). However, you can do so if you wish. For example, you can perform the above-described procedure for U up to 8kV (with $N_s = 10$). If you try to perform it for $U = 9kV$, the following information will appear on the screen:

```
s = 0.0000 (initial approximation)
      Tcen=1908      Tedge=1908
```

```
Unable to compute the near-cathode plasma layer for
```

```
Tw = 4762K,      U = 9000.0000V
```

```
Fine-tune the subroutine CVC_U or do not call it for these
```


conditions (which are, presumably, extreme). The code terminated.

The code successfully generated an initial approximation, however the initialization of iterations failed and the iteration process has not even started. The problem may be fixed by fine-tuning the code, however values of U that high are not of any interest anyway.

The current-voltage characteristic $U(I)$ of the diffuse discharge is U-shaped, i.e., falls at low currents, then passes through a minimum and starts rising. The above explains how to generate a starting-point solution for a low-current section of the diffuse mode, which is characterized by a falling current-voltage characteristic. In order to signal the code that you want to generate a starting-point solution for the high-current section, which is characterized by a rising current-voltage characteristic, it is sufficient to enter minus before N_s . However, the number of steps in which the starting-point solution is generated should be quite high in this case. For example, setting $N_s = -50$ for $U = 15V$ results in the interruption of the code after the first iteration at $s = 0.08$ with the diagnostics

```
The current iteration has been completed. The calculated
temperature of some points of the lateral surface
exceeds 6500K. The code terminated.
```

Setting $N_s = -100$ will fix the problem. Note that the number of steps in which the starting-point solution for the high-current section is generated may be decreased if U is as low as possible: for $U = 12V$, for example, $N_s = -20$ is fine.

If the lateral surface of the cathode is electrically and thermally insulated, the above-described (automatic) procedure generates a starting-point solution in one step (at which just one iteration is sufficient). Therefore, if parameter `Insulated lateral surface` is set equal to `.t.`, then you should set $N_s = 1$ for the low-current branch of the diffuse mode or $N_s = -1$ for the high-current branch; the code will issue an error message and terminate if the specified value of N_s exceeds 1 or is below -1.

Current transfer from high-pressure plasmas to thermionic cathodes can occur not only in the diffuse mode, but also in the spot mode. The generation of starting-point solutions for different branches of the axially symmetric spot mode is explained in Section 4.

3. Step 3: Performing simulations

Perform once again the procedure described in the beginning of Section 2, thus generating the starting-point solution for $U = 15V$ on the low-current section of the diffuse mode, and then go to the tab entitled `Step 3`. In order to start the simulations, you need to specify the near-cathode voltage (in volts) corresponding to the starting point, U_0 , the near-cathode voltage (in volts) corresponding to the last point, U_{fin} , and the number N_U of steps between the starting and last points. You need also to indicate whether you want to store the solution corresponding to the last point as a starting point for further calculations (which means that the currently existing starting-point solution will be overwritten, so you will have to generate it once again should you need it). Enter $U_0 = 15$, $U_{fin} = 13$, $N_U = 2$, `Yes`, respectively. Make sure that there is -1 in the field `Number of the bifurcation point at this stage`, $N_b = -1$, and press the button `Start simulations`. The following information appears in the window **Program output**:

	U (V)	I (A)	Tcen (K)	Tedge (K)
1	15.0000	20.49	3281	3419
1				

```

2
3
4
14.0000  24.84      3329   3466
1
2
3
4
13.0000  31.49      3391   3528

```

The simulations have been completed.

You can see that there was 1 iteration for $U = 15\text{V}$, 4 iterations for $U = 14\text{V}$ and 4 iterations for $U = 13\text{V}$. The arc current is 20.49A at $U = 15\text{V}$, 24.84A at $U = 14\text{V}$ and 31.49A at $U = 13\text{V}$. Note that the role of an initial approximation at $U = 15\text{V}$ was played by the starting-point solution, i.e., by an exact solution, which is why just one iteration was sufficient for convergence. The role of the initial approximation at each of the subsequent values of U was played by the preceding solution, so several iterations were necessary. The arc current I increases with decreasing U , so the current-voltage characteristic is falling as it should.

The following integral characteristics of the discharge for all values of U for which simulations were performed (15V, 14V, 13V) appear in the window **Integral characteristics**:

- arc current I ,
- heat Q_c removed by thermal conduction to the cathode base,
- heat Q_{rad} radiated by the cathode surface,
- temperatures T_{cen} and T_{edge} at the center and at the edge of the front surface of the cathode.

It may be a good idea to copy right now this information to your computer through COPY and PASTE commands. Note that the integral energy fluxes Q_c and Q_{rad} do not appear in cases where parameter `Insulated lateral surface` is set equal to `.t.`

Press the button `View distributions`. You can see in the window **Distributions** distributions along the cathode surface of the following parameters:

- surface temperature T_w ,
- density j of the electric current from the plasma to the cathode surface.

The distributions correspond to the last value of U calculated and are presented as functions of the distance $r+z$, where r and z are cylindrical coordinates with the origin at the center of the front surface of the cathode and with the z -axis directed from the front surface into the cathode body; see Figure 1. In other words, $r+z$ is the distance from the center of the front surface of the cathode measured along the generatrix. In accordance with this choice, $\{r \leq R, z = 0\}$ is the (circular) front surface of the cathode while $\{r = R, z \geq 0\}$ is the (cylindrical) lateral surface (here R is the radius of the cathode, which is set equal to 1 mm in these calculations). Thus, the range $0 \leq r+z \leq R$ corresponds to the front surface while the range $r+z \geq R$ corresponds to the lateral surface. If you need these data, you should copy them to your computer through COPY and PASTE commands.

Note that these distributions may be used as boundary conditions for differential equations describing the arc bulk and thus allow one to obtain a model of the arc on the whole with a self-consistent description of the arc-cathode interaction [4-6].

Set the starting-point voltage U_0 equal to 13V, the last-point voltage U_{fin} equal to 7V, and the number N_U of steps between the starting and last points equal to 60. Press the button `Start simulations`. You can see that the iterations converged at $U \geq 10.1\text{V}$ and diverged at $U = 10.0\text{V}$. Hence, the minimum of the current-voltage characteristic of the diffuse mode is somewhere around 10.1V. Set $U_0 = 13\text{V}$, U_{fin}

= 10.1V, $N_U = 3$, and perform the simulation. After it has been completed, an initial approximation for $U = 10.1V$ is available. Now you can start simulations from $U = 10.1V$ with a smaller step in U (0.01V, for example) in order to determine the minimum point with a higher precision.

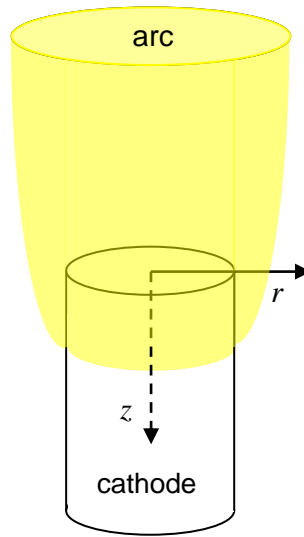


Figure 1. The system of coordinates.

The above-described simulations refer to the low-current section of the diffuse mode. Simulations on the high-current section are performed in a similar way. Go to the tab `Step 2` and generate the starting-point solution for $U = 12V$ on the high-current section with $N_s = -20$, then go to the tab `Step 3`, set $U_0 = 12V$, $U_{fin} = 13V$, $N_U = 1$, and perform the simulation. The iterations will diverge. If, however, you set $N_U = 4$, then the iterations converge. This example shows that the step in U on the high-current section of the diffuse mode must in a general case be lower than on the low-current section. The reason is that a solution on the high-current section is more sensitive with respect to U than a solution on the low-current section or, as one can say, that the (rising) current-voltage characteristic $U(I)$ of the high-current section is appreciably flatter than the (falling) current-voltage characteristic of the low-current section.

Part B. Advanced features

4. Finding multiple solutions describing different modes of current transfer

4.1. Encountering multiple solutions

Return to the tab `Step 1` and specify once again our “standard” conditions (those described at the end of Section 1) with the difference that this time the cathode radius should be set equal to 2mm. Go to `Step 2` and generate the starting-point solution for $U = 15\text{V}$ with $N_s = 1$. Running the code in the tab `Step 3` with the settings $U_0 = 15\text{V}$, $U_{fin} = 15\text{V}$, $N_U = 1$, $N_b = -1$, N_o , you will find that the considered solution corresponds to the arc current of $I = 82.80\text{A}$.

Now return to `Step 2` and generate the starting-point solution for $U = 12\text{V}$ with $N_s = 2$. Move from 12V to 15V by running the code in the tab `Step 3` with the settings $U_0 = 12\text{V}$, $U_{fin} = 15\text{V}$, $N_U = 3$, $N_b = 0$, N_o . You see that the arc current at $U = 15\text{V}$ this time equals 77.14A. Thus, you have found two different solutions for the same near-cathode voltage of 15V, one with the arc current of 82.80A and another with 77.14A. The difference between the current values is not large, however if you plot distributions of the surface temperature, as shown in Figure 2, you will find them substantially different.

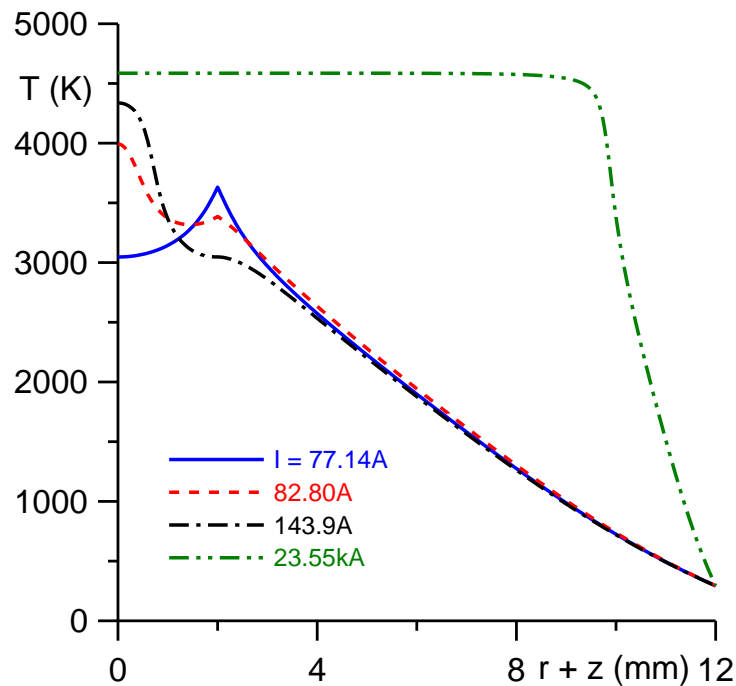


Figure 2. Distributions of temperature of the cathode surface described by different solutions existing for $U = 15\text{V}$. W cathode, $R = 2\text{mm}$, $h = 10\text{mm}$, $T_{col} = 293\text{K}$, Ar plasma, $p = 1\text{bar}$.

This example illustrates the most important feature of the model of nonlinear surface heating: the model can provide more than one steady-state solution for the same conditions, with different solutions describing different modes of current transfer. Current-voltage characteristics of different steady-state modes that exist under the considered conditions are shown in Figure 3 (taken from [1]). Five of the steady states existing at $U = 15\text{V}$ (those with spots at the edge of the front surface of the cathode) are 3D. The remaining four states are axially symmetric and can, at least in principle, be found by means of this code:

- A state belonging to the low-current section, $I \leq 0.9\text{kA}$, of the diffuse mode, which is characterized by a falling current-voltage characteristic (this state has already been computed and is shown in Figure 2 by the solid (blue) line);
- A state belonging to the high-current section, $I \geq 0.9\text{kA}$, of the diffuse mode, which is characterized by a rising current-voltage characteristic and is beyond the current range shown in Figure 3;
- A state belonging to the low-voltage (left-hand) branch of the mode with a spot at the center of the front surface of the cathode (this state has already been computed and is shown in Figure 2 by the dashed (red) line);
- A state belonging to the high-voltage (right-hand) branch of the mode with a spot at the center.

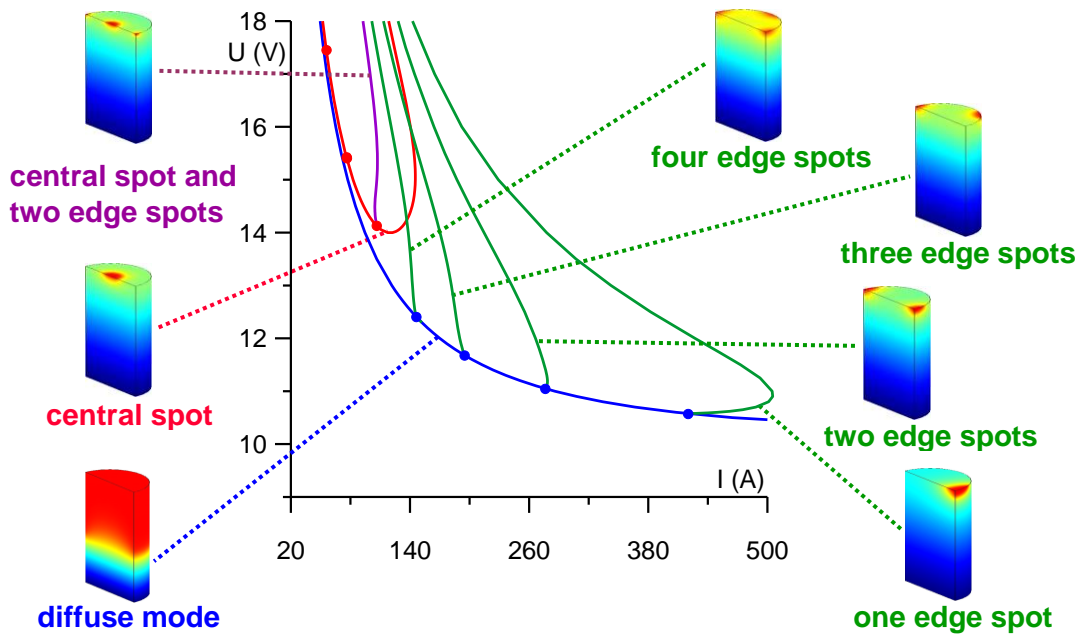


Figure 3. Current-voltage characteristics of different steady-state modes and typical distributions of the temperature of the cathode surface associated with each mode. Circles: bifurcation points. W cathode, $R = 2\text{mm}$, $h = 10\text{mm}$, $T_{col} = 293\text{K}$, Ar plasma, $p = 1\text{bar}$.

4.2 Finding different axially symmetric solutions

The aim of this Section is to learn how solutions describing each of the four above-indicated axially symmetric steady states can be computed. It was clearly demonstrated by the above example that the convergence of iterations to one or the other of the existing solutions is governed by a choice of the initial approximation. Therefore, we need to find out what initial approximations are appropriate to force the iterations to converge to each of the four solutions.

The following procedure is employed by the code when the field `Generating starting-point solution automatically` in the tab `Step 2` has been activated. First, an initial approximation is generated by means of the analytical solution [7] describing the diffuse mode on a cylindrical cathode with a thermally insulated lateral surface. This solution is 1D: the temperature T inside the cathode varies linearly along the cathode axis and does not vary in the transversal directions; in other words, T is a linear function of the coordinate z and does not depend on r . [Strictly speaking, what varies linearly

with z is the heat flux potential, which is a quantity related to T by the expression $\psi = \int \kappa(T) dT$, where κ is thermal conductivity of the cathode material. However, thermal conductivity of tungsten does not vary much at $T \geq 1000\text{K}$, thus T also varies approximately linearly with z .] The generated 1D solution corresponds to the specified U and belongs to the low- or, respectively high-current section of the diffuse mode depending on the sign of N_s . After the 1D solution has been generated, the code starts simulations with a gradual elimination of the thermal insulation on the lateral surface, which corresponds to an increase in the transition parameter s mentioned in Section 2 from 0 to 1.

It could be expected that, starting from a solution describing the diffuse mode on a cylindrical cathode with a thermally insulated lateral surface and then gradually removing the thermal insulation, one would arrive at a solution describing the diffuse mode on a cathode with an active lateral surface. This is true in most cases. An example of a case where this is untrue is the 2mm-radius cathode being treated in the present Section when it operates in the diffuse mode at high enough voltages (low currents): the starting-point solution generated for $U = 15\text{V}$ with $N_s = 1$ (the line corresponding to $I = 82.80\text{A}$ in Figure 2) describes a temperature distribution with maxima both at the center of the front surface of the cathode and at the edge, and such distributions are associated with the low-voltage branch of the mode with a spot at the center rather than with the diffuse mode [8]. In other words, the transition from the diffuse mode on a cathode with the insulated lateral surface ends up at the low-voltage branch of the central-spot mode on the cathode with the active lateral surface rather than at the diffuse mode. This difficulty cannot be overcome by simply increasing the number of steps in which the thermal insulation is removed: the transition still ends up at the low-voltage branch of the central-spot mode even if you set, e.g., $N_s = 100$. A simple way to overcome this difficulty is to start from the diffuse mode on the cathode with an insulated lateral surface at U below approximately 14V, where the central-spot mode does not exist as seen in figure 3. Then the transition process will, presumably, end up at the diffuse mode. After this, one can move to $U = 15\text{V}$, and remaining on the diffuse mode during this move may be ensured by choosing a small enough step in U . This approach was employed in the beginning of the present Section and works nicely: the temperature distribution obtained, which is represented by the solid (blue) line in Figure 2, has a maximum at the edge of the front surface of the cathode and is indeed associated with the diffuse mode.

As far as the high-current section of the diffuse mode is concerned, moving from the cathode with an insulated lateral surface to the cathode with an active lateral surface poses no problems, provided that the thermal insulation is removed in small steps. In the example being considered, you can obtain, by setting $U = 15\text{V}$ and $N_s = -50$, a solution which corresponds to the arc current of $I = 23.55\text{kA}$ and is shown in Figure 2 by the two dots-dashed (dark green) line. The temperature is very high and constant (“saturated”) along the front surface and the most part of the lateral surface; a feature which is characteristic for the high-current section of the diffuse mode [8]. This temperature distribution is very different from the (1D) solution for the cathode with the insulated lateral surface, in contrast to the temperature distribution associated with the low-current section (the blue line in Figure 2), which varies along the lateral surface approximately linearly in the axial coordinate and is in this respect similar to the 1D solution. This explains a large number of steps needed in order to move from the high-current section of the diffuse mode on the cathode with an insulated lateral surface to the high-current section of the diffuse mode on the cathode with an active lateral surface.

Thus, solutions describing the diffuse mode can be obtained with the use of the built-in (1D) initial approximation, except in the case of wide cathodes and low currents. It is interesting to note that although the difficulties arising at low currents on wide cathodes have been known since this code was developed in 2001, they were paid attention and understood only seven years later [9], after similar difficulties have been encountered in the modeling of glow cathodes. The reason of these difficulties is an exchange of branches between the diffuse and central-spot modes, which occurs when the thermal insulation becomes imperfect thus breaking the transcritical bifurcation through which the first axially

symmetric spot mode on a cathode with an insulated lateral surface branches off from the diffuse mode. Due to this exchange of branches, the transition between diffuse modes on wide arc cathodes with insulated and active lateral surfaces at low enough currents is discontinuous and a gradual elimination of thermal insulation from a wide cathode operating in the diffuse mode at a low enough current leads to the low-voltage branch of the central-spot mode rather than to the diffuse mode.

Let us proceed to finding initial approximations appropriate for simulations of each of the two branches of the central-spot mode. Go to the tab `Step 1`, change the cathode radius to 1.5mm, then go to the tab `Step 2` and generate a starting-point solution for $U = 17V$ from the built-in initial approximation with $N_s \geq 3$. The obtained solution belongs to the low-current section of the diffuse mode as expected. You will obtain another solution by generating a starting-point solution for $U = 17V$ from the built-in initial approximation with $N_s = 1$. A third solution can be obtained by generating a starting-point solution for $U = 40V$ from the built-in initial approximation with $N_s \geq 15$ and then moving to $U = 17V$. Comparing the second and third solutions, you can conclude that they belong to the high- and, respectively, low-voltage branches of the central-spot mode. It should be stressed that while such way of finding a solution for the low-voltage branch is mathematically justified (a gradual elimination of thermal insulation from a wide cathode operating in the diffuse mode at a high enough voltage leads to the low-voltage branch of the central-spot mode), the convergence of the iterations to the high-voltage branch is a mere coincidence.

This example shows that solutions describing both branches of the spot mode in some cases can be obtained with the use of the built-in initial approximation. On the other hand, a systematic and reliable approach is desirable. More control over the choice of initial approximation is needed for such approach to be possible, and the code provides the possibility of specifying an initial approximation manually. This approximation is governed by three parameters: the temperatures at the center and at the edge of the front surface of the cathode, T_{cen} and T_{edge} , and a “spot radius” R_s . On the front surface of the cathode, the temperature varies between T_{cen} and T_{edge} exponentially in r , or, more precisely, the heat flux potential varies exponentially:

$$\psi = C_1 \exp(-r/R_s) + C_2,$$

where C_1 and C_2 are constants defined by the requirements

$$T = T_{cen} \text{ at } r = 0, \quad T = T_{edge} \text{ at } r = R$$

(we remind that R is the cathode radius). On the lateral surface, the heat flux potential varies linearly in z between values corresponding to T_{edge} and T_{col} .

The simplest strategy is the following. The temperature inside the spot on the high-voltage branch is higher than on the low-voltage branch, but in any case the temperature cannot exceed the above-mentioned “saturated” value [8], which for $U = 15V$ equals approximately 4586K. In order to arrive at the high-voltage branch, one can set T_{cen} equal to or somewhat above this value, assume a moderate value for T_{edge} , and assume the linear variation of the temperature (heat flux potential) along the front surface, which amounts to attributing to R_s a value considerably exceeding R . In order to arrive at the low-voltage branch, one can set T_{cen} below the “saturated” value.

Let us check whether this strategy works. Restore $R = 2mm$, go to the tab `Step 2` and set $U = 15V$. Choose `Yes` in the field `Generating starting-point solution from a manually-defined initial approximation` and set $T_{cen} = 4586K$, $T_{edge} = 3000K$, $R_s = 1m$. Make sure that there is `0` in the field `Damper` and press `Start`. The iterations rapidly converge; only 7 iterations were needed. The obtained solution corresponds to the current of 143.9A and to the temperature distribution that is shown in Figure 2 by the dot-dashed (black) line; as expected, this distribution is typical for the high-voltage branch of the spot mode. You can check that the convergence of iterations is

not very sensitive with respect to T_{cen} as long it is above the “saturated” temperature: you will arrive to the same solution in the same number of iterations if you set, e.g., $T_{cen} = 5000\text{K}$. If, however, you set $T_{cen} = 4500\text{K}$, i.e., below the saturated value, the iterations will diverge.

Insert the value of 0.5 into the field `Damper` and try once again the value $T_{cen} = 4500\text{K}$. This time, the iterations converge, although rather slowly (37 iterations). This example demonstrates the effect of damping, which is an important trick that is frequently critical to attaining convergence of iterations in nonlinear problems of very different nature. Damping amounts to averaging the result of each iteration with the result of the previous one, with weights equal to $(1-d)$ and, respectively, d , where d is an adjustable parameter, the so-called damper. d varies between 0 (no damping) and values close to 1 (heavy damping; only small changes of a solution between two successive iterations are allowed). The optimal strategy, which is usually implemented in commercial nonlinear solvers, is to employ heavy damping at the initial stage while the iterations are far from convergence, and then reduce and eventually remove damping as the iterations approach convergence. Such sophistication is not warranted in the problem being considered, so the code does not change the damper in the course of iterations.

Change T_{cen} to 4400K, leaving the other fields unaltered ($U = 15\text{V}$, $T_{edge} = 3000\text{K}$, $R_s = 1\text{m}$, $d = 0.5$), and run the code. The iterations still converge, although a little bit slower (40 iterations). There is no convergence with $T_{cen} = 4300\text{K}$ and $T_{cen} = 4200\text{K}$. The iterations converge once again at $T_{cen} = 4100\text{K}$, however to a different solution, namely, to the one that describes the low-voltage branch of the central-spot mode and is shown in Figure 2 by the dashed (red) line. You can arrive at the same solution also with $T_{cen} = 4000\text{K}$ and $T_{cen} = 3900\text{K}$. Thus, the above-described simplest strategy has proved working and efficient: it is sufficient to choose T_{cen} close to the “saturated” value or above it, assume a moderate value for T_{edge} , and the linear variation of the temperature (heat flux potential) along the front surface in order to arrive at the high-voltage branch; then T_{cen} is gradually decreased until the low-voltage branch has been attained.

Of course, other possibilities exist as well; for example, if you reduce T_{cen} further, you will see that the iterations do not converge at $T_{cen} = 3800\text{K}$, converge to the high-voltage branch of the spot mode at $T_{cen} = 3700\text{K}$, do not converge at $T_{cen} = 3600\text{K}$, converge to the low-voltage branch at $T_{cen} = 3500\text{K}$, and do not converge at lower T_{cen} . One can conclude that although the islands of convergence do exist, they are narrow and their occurrence appears irregular, so this possibility is more difficult to exploit than the above-described strategy.

4.3. Simulating the spot mode

The only one spot mode that is stable is the 3D mode with a spot at the edge; all the other spot modes are unstable and cannot be observed in the experiment. The latter applies, in particular, to the axially symmetric spot modes that can be simulated by means of this code. However, the physics of 3D and axially symmetric spots is the same, and the difficulties arising in their calculation are very similar. This is why the simulation of axially symmetric spot modes by means of this code is of considerable methodological interest.

The code employs a uniform rectangular numerical grid of the second order of accuracy. By default, the grid contains 300 steps in the axial direction and 30 steps in the radial direction, which corresponds to 301 equally spaced knots on the axis and 31 knots on the front surface. This grid is sufficiently accurate for calculations of the diffuse mode. One can see from figure 2 that the grid is adequate also for the spot mode at low U , where the spot is not very well pronounced. However, this is not necessarily true at higher U . Generate the starting-point solution on the high-voltage branch of the spot mode at $U = 15\text{V}$ as described in the preceding Section, then move to $U = 25\text{V}$ by running the code in the tab `Step 3`

with the settings $U_0 = 15\text{V}$, $U_{fin} = 25\text{V}$, $N_U = 10$, $N_b = 0$, `No`. You can see that the spot has shrunk considerably and the temperature variation over one step of the grid reaches nearly 500K; the grid is clearly inadequate. This example shows that as U increases, the spot shrinks and the default numerical grid employed by the code turns insufficient.

Thus, one may need to refine the grid in some cases. To this end, go to `Step 2`, choose `No` in the field `The default numerical grid will be used (recommended)`, and specify desired numbers of steps of the grid in the axial and radial directions. Note that this option must be used with caution, since an increase of the number of steps (especially in the radial direction) can make the code significantly slower. Note also that after the number of steps has been changed, the starting-point solution, if it exists, becomes inapplicable and must be generated again. For this reason, there is no separate `Submit` button and the changes are activated by pressing the button `Start` in the group `Generating starting-point solution automatically` or the button `Start` in the group `Generating starting-point solution from a manually-defined initial approximation`.

The most difficult point in simulations of the spot mode is that if iterations diverge, you have no means to know what the reason is: the initial approximation may be inadequate, or the grid may be inadequate, or the spot mode may simply not exist under these conditions. The uncertainty may be reduced if an initial approximation is used that is not defined manually as described in the preceding Section, but rather represents a solution describing the desired mode under close conditions. Then one can ensure that the initial approximation is good enough and can check whether the grid is adequate. If, nevertheless, the iterations do not converge, then the most likely reason is the inexistence of the solution being sought.

Consider, for example, calculation of the high-voltage branch of the spot mode on a cathode of the radius of 1.5mm. The best way is to start with a solution describing the high-voltage branch of the spot mode on a 2mm-radius cathode. Generate this solution as described in the preceding Section (for $U = 15\text{V}$), then go to the tab `Step 1` and change the cathode radius to 1.5mm, then run the code in the tab `Step 3` with the settings $U_0 = 15\text{V}$, $U_{fin} = 15\text{V}$, $N_U = 1$, $N_b = 0$, `Yes`. The iterations diverge. Is the initial approximation not good enough? Try changing the radius not in one but in several steps. You can find that the iterations diverge when you try to move from $R = 1.73\text{mm}$ to $R = 1.72\text{mm}$, although there is no problem in switching in one step from $R = 2\text{mm}$ to $R = 1.73\text{mm}$. Thus, a better initial approximation does not help; the problem must be elsewhere. By plotting the surface temperature distribution at $R = 1.73\text{mm}$, you can check that the grid also is not a problem. Hence, the most likely reason of divergence of the iterations is the inexistence of the spot mode.

It is legitimate in this situation to hypothesize that the decrease of the cathode radius from 2mm to 1.5mm has caused a shift of the spot mode in the direction of higher voltages, so $U = 15\text{V}$ is below the minimum voltage of this mode at $R = 1.5\text{mm}$. Generate once again the solution describing the high-voltage branch of the spot mode on a 2mm-radius cathode as described in the preceding Section (for $U = 15\text{V}$), then move to $U = 25\text{V}$ by running the code in the tab `Step 3` with the settings $U_0 = 15\text{V}$, $U_{fin} = 25\text{V}$, $N_U = 10$, $N_b = 0$, `Yes`. Now reduce R to 1.5mm and run the code in the tab `Step 3` with the settings $U_0 = 25\text{V}$, $U_{fin} = 25\text{V}$, $N_U = 1$, $N_b = 0$, `Yes`. The iterations converge: you have got a solution for $R = 1.5\text{mm}$; the above hypothesis was correct. Now move into the direction of lower U . You will find that the iterations stop converging during the move from $U = 16.23\text{V}$ to $U = 16.22\text{V}$. The obtained current-voltage characteristic is depicted by the right-hand side line in Figure 4, and it seems that $U = 16.23\text{V}$ represents its point of minimum.

You can use the same procedure for finding the low-voltage branch of the spot mode on a 1.5mm-radius cathode, the only difference being that the departure point will be a solution describing the low-voltage branch of the spot mode on a 2mm-radius cathode and not the high-voltage branch. The obtained current-voltage characteristic is depicted by the left-hand side line in Figure 4, and the iterations again stop converging during the move from $U = 16.23\text{V}$ to $U = 16.22\text{V}$. One can conclude that $U = 16.23\text{V}$ is indeed the point of minimum. There is a small gap between the two branches of the current-voltage characteristic, which can be reduced by decreasing U in smaller steps.

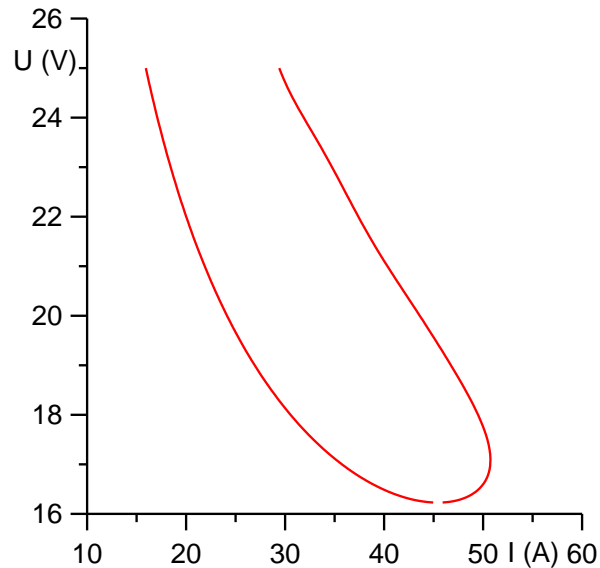


Figure 4. Current-voltage characteristic of the mode with a spot at the center. W cathode, $R = 1.5\text{mm}$, $h = 10\text{mm}$, $T_{col} = 293\text{K}$, Ar plasma, $p = 1\text{bar}$.

5. Finding the limit of stability of the diffuse mode and bifurcation points positioned on axially symmetric modes of current transfer

In addition to the calculation of different axially symmetric steady-state modes of current transfer, the code offers the possibility of finding bifurcation points at which other modes, in particular, 3D spot modes, branch off from the axially symmetric modes. (These points are represented in Figure 3 by circles.) The importance of information on bifurcation points is two-fold. First, it facilitates finding 3D solutions. Second, a bifurcation point at which the steady-state 3D mode with a spot at the edge of the front surface branches off from the steady-state diffuse mode represents the limit of stability of the diffuse mode: the diffuse mode is stable at arc higher currents and unstable at lower currents.

Specify once again the conditions described at the end of Section 1 with an exception that $R = 2\text{mm}$. Generate the starting-point solution for $U = 12\text{V}$ corresponding to the low-current section of the diffuse mode by running the code in the tab Step 2 with the settings $U = 12\text{V}$, $N_s = 2$. Go to the tab Step 3. Up to now, there was -1 in the field Number of the bifurcation point, $N_b = -1$, which signaled the code that no bifurcation point is sought and the relevant module of the code should not be executed. Now you should set N_b equal to the number of the bifurcation point which you want to find. If you want to find a bifurcation point at which a 3D mode with one, or two, or three *etc* spots at the edge of the front surface of the cathode branches off from the axially symmetric mode being considered, then you should set N_b equal to 1, or 2, or 3 *etc*. Set $N_b = 0$ if you simulate the (1D) diffuse mode on a cathode with an electrically and thermally insulated lateral surface and wish to find a bifurcation point at

which an axially symmetric spot mode branches off (or *vice versa*: if you simulate an axially symmetric spot mode and wish to find a bifurcation point at which the 1D diffuse mode branch off). If you are interested only in the limit of stability of the diffuse mode and not in branching of spot modes, set $N_b = 1$.

Set $U_0 = 12$, $U_{fin} = 10$, $N_U = 20$, $N_b = 1$, No, and press the button `Start simulations`. In addition to the information described in Section 3, the output windows **Program output** and **Integral characteristics** now comprise Del , which is the determinant of a finite-difference problem corresponding to the differential eigenvalue problem for axially symmetric and 3D perturbations of the axially symmetric solution; see problem (12), (13) of the paper [10]. The change of sign of Del signals the bifurcation point. Thus, in order to find the bifurcation point you should first choose an interval $[U_0, U_{fin}]$ wide enough in order to ensure that Del does change sign inside this interval, and then localize this change of sign to a required accuracy. Note that the sign of Del on the diffuse mode normally changes from plus to minus as the arc current decreases.

In the case being treated, the change of sign of Del has occurred between $U = 10.6\text{V}$ and $U = 10.5\text{V}$. Investigating this interval with successively decreasing steps in U , you will find that under the considered conditions the limit of stability of the diffuse mode or, equivalently, the bifurcation point at which the steady-state 3D mode with a spot at the edge branches off is $I = 416.3\text{A}$ ($U = 10.5917\text{V}$). This is the point represented by the rightmost blue circle in Figure 3. Changing N_b to 2, you will find the next bifurcation point: $I = 273.7\text{A}$ ($U = 11.0704\text{V}$) *etc.*

The bifurcation points positioned on the axially symmetric spot mode can be found in a similar way. Setting, for example, $N_b = 2$, you will find the bifurcation point $I = 105.8\text{A}$ ($U = 14.1585\text{V}$). This is the bifurcation point represented by the lowest red circle in Figure 3.

Part C. If the code runs on your computer

The above refers to the case where you run the code via Internet on a server of Universidade da Madeira. If you have received from us a Windows version of the code and run it on your computer, the following changes should be introduced into the above.

The standard distribution of the code includes, apart from this file (tutorial.pdf), ten files: f0.exe, f1.exe, input.dat, parameters_of_grid.dat, parameters_of_run.dat, cmd.bat, intinF0.txt, intinF1.txt, run_F0_input_from_file.bat, run_F1_input_from_file.bat. Copy these ten files into a working folder. Click the file cmd.bat. A DOS window will open. It is convenient to have this window side by side with the working folder, because you will need to work in both: to edit input files (any ASCII editor can be used) in the working folder and to perform simulations in the DOS window.

Section 1

The input parameters are specified by editing the files input.dat, parameters_of_grid.dat, and parameters_of_run.dat. The files input.dat and parameters_of_grid.dat are self-explaining. Note that the value 0.D0 in the 9th and/or 10th lines of the file input.dat (user-specified values of the work function and the pre-exponential factor in the Richardson-Dushman formula) instructs the code to use values of the work function and the pre-exponential factor for the given cathode material from the internal database. We remind that after the parameters in the file parameters_of_grid.dat have been changed, the starting-point solution, if it exists, becomes inapplicable and must be generated again.

The logical value in the first line of the file parameters_of_run.dat indicates whether prompts will, or not, appear on the screen during the runtime. The number in the second line defines the maximum time of continual operation of the code (in seconds). The default value is 3600, i.e., one hour. Normally, you do not have to change this value since you can interrupt the code at any moment by pressing `Ctrl C`; and in any case the code takes much less time to run – provided, of course, that the input parameters are appropriate. (Smaller values of the maximum time are required when the code is operated via Internet from an UMa server.)

The third and fourth lines of the file parameters_of_run.dat are not used at the step 2, however they have to be specified before proceeding to step 3. The content of the third line is described in Section 5. The content of the last line (`printq`) defines whether the output file distrib.dat will comprise, in addition to the distributions specified in the description of the window **Distributions** given in Section 3, also the calculated distributions along the cathode surface of the following parameters:

- temperature T_e of electrons in the near-cathode plasma layer,
- density q of the net energy flux to the cathode surface ($q = q_p - q_r$, where q_p is the density of plasma-related energy flux and q_r is the density of radiation losses of energy by the cathode surface).

In the case where parameter `Insulated lateral surface` is set equal to `.t.`, value of `printq` defines also whether the output file int_char.dat will comprise the integral energy fluxes Q_c and Q_{rad} .

Section 2

Switch to the DOS window and type `f0` (and press Enter). You will be prompted to specify first the near-cathode voltage U (in volts) for which you want a starting-point solution to be generated and then the number N_s of steps in which this solution will be generated. If you enter N_s different from zero, the code will generate a starting-point solution automatically. If you enter $N_s = 0$, the code will assume that you wish to generate a starting-point solution from a manually-defined initial approximation and will prompt you to enter T_{cen} , T_{edge} , R_s , and $Damper$. After the code has been successfully completed, the starting-point solution is stored in the file `initial.dat` created in the working folder.

Instead of supplying the data to the code interactively (on being prompted by the code during the runtime), you can supply the same data by editing the file `intinF0.txt`. (A mnemonic rule: `intinF0` stands for INTERactive INput for F0.) This option is especially handy in cases where you generate the starting-point solution manually. While using this option, you may wish to switch off the prompts (which is done by entering `.f.` into the first line of the file `parameters_of_run.dat`). After you are done with editing the files `intinF0.txt` and `parameters_of_run.dat`, switch to the DOS window and launch the code by typing `"f0.exe" < intinF0.txt`. Alternatively, you can launch the code by clicking the file `run_F0_input_from_file.bat` in the working folder, in which case you do not need to open the DOS window manually since it will be opened and closed automatically.

Section 3

Make sure that the parameters in the last two lines of file `parameters_of_run.dat` are right. Switch to the DOS window and type `f1` (and press Enter). The program will successively prompt you to enter the near-cathode voltage U_0 (in volts) which corresponds to the starting point, the near-cathode voltage U_{fin} (in volts) which corresponds to the last point, and the number N_U of steps between the initial and final points. At the end of the modeling, the program will ask whether you want to use the solution corresponding to the last point as a starting point for further calculations. Instead of supplying the data to the code interactively, you can do so by editing the file `intinF1.txt`, after which the code is launched by typing `"f1.exe" < intinF1.txt` or by clicking the file `run_F1_input_from_file.bat` in the working folder.

The two files with simulation results, `int_char.dat` and `and_distrib.dat`, are generated in the working folder. Note that these files are overwritten in each subsequent simulation, so it may be a good idea to immediately copy contents of both files somewhere else through COPY and PASTE commands.

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